American University of Beirut
Math 204
Quiz I (Fall 2013)

Time 50 minutes .

Name: ____________________ **ID#:** ____________________

Circle your problem solving section number below:

- **Instructor: Ms Joumana Tannous**

Section 1 @ 1:00 M **Section 2 @ 12:00 M** **Section 3 @ 4:00 M**

- **Instructor: Mrs Maha Itani-Hatab**

Section 4 @ 11:00 Tu **Section 5 @ 8:00 Tu** **Section 6 @ 12:30 Tu**

- **Instructor: Ms. Michella Bou Eid**

Section 7 @ 12:30 Th **Section 8 @ 2:00 Th** **Section 13 @ 5:00 Th**

- **Instructor: Ms Najwa Fuleihan**

Section 10 @ 8:00 Tu **Section 11 @ 9:30 Tu** **Section 12 @ 11:00 Tu**

Solution

- **Answer table for Part I**

1	2	3	4	5	6	7	8	9	10
A	C	B	D	C	A	D	B	C	A

# of correct answers : ----------------------- # of wrong answers : -------------------------							Grade of Part I 35%	
1.	2.	3.	4.	5.	6.	7.	Grade of Part II 65%	Final Grade

<u>Part I</u> : 10 multiple choice questions with 3.5 points for each correct answer and 0.5 penalty for each wrong answer.

Circle the correct answer then, copy <u>your answers as a, b, c or d on the table provided on page 1</u>:

Given the matrices $A=\begin{pmatrix}1 & 2\\ 3 & 1\\ -1 & 0\end{pmatrix}$, $B=\begin{pmatrix}1 & 2\\ 0 & 4\end{pmatrix}$ and $C=\begin{pmatrix}1 & -2 & 1\\ 3 & 0 & 1\end{pmatrix}$

1. $4\left(B^{-1}\right)^{T}+\left(CAC\right)^{0}$ is ....

a. undefined **b.** $\begin{pmatrix}5 & 0\\ -2 & 2\end{pmatrix}$ **c.** $\begin{pmatrix}5 & -2\\ 0 & 2\end{pmatrix}$ **d.** $\begin{pmatrix}4 & -2\\ 0 & 1\end{pmatrix}+1$

2. AC is .....

a. undefined **b.** $\begin{pmatrix}7 & 2 & 1\\ -2 & -6 & -2\\ 1 & -2 & 1\end{pmatrix}$ **c.** $\begin{pmatrix}7 & -2 & 3\\ 6 & -6 & 4\\ -1 & 2 & -1\end{pmatrix}$ **d.** $\begin{pmatrix}-6 & 0\\ 2 & 6\end{pmatrix}$

3. Let $E=\begin{pmatrix}-1 & 0\\ -4 & x\end{pmatrix}$, the value of x for which $\left(EI^{-1}+2B^{T}\right)=I$ is .....

a. -1 **b.** -7 **c.** 0 **d.** -4

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4. The values of p for which the system $\begin{pmatrix}2 & -p\\ 4 & -4\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}3\\ -2\end{pmatrix}$ admits a unique solution are ...

a. $p\neq -2$ **b.** $p\neq -1$ **c.** $p\neq 1$ **d.** $p\neq 2$

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5. . If $A=\begin{pmatrix}1 & 2 & 3\\ 0 & 1 & 5\\ 0 & 9 & 2\end{pmatrix}$ and $B=\begin{pmatrix}-3 & 1 & 1\\ 0 & 1 & 5\\ 0 & 0 & -2\end{pmatrix}$ then $\det\left(A\,B\,A^{-1}\right)$ is ......

a. -4 **b.** -6 **c.** 6 **d.** 4

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6. Given $A=\begin{pmatrix}2&1&0&2\\8&5&2&9\\0&3&0&0\\4&7&0&5\end{pmatrix}$ then the cofactor c_{32} is ......

a. -4 b. -12 c. 12 d. 4

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7. Let A be a (4×3) matrix, B a (3×3) matrix and M a (3×4) matrix. If O is the zero matrix and the operation $\left(AB+(BM)^{T}\right)O$ can be performed then the dimension (size) of O is ......

a. undefined b. 4×3 c. 3 d. $3\times n$ where n is a positive integer

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8. Let $\begin{pmatrix}y+4\\x+3\end{pmatrix}-2\begin{pmatrix}2-x\\3-y\end{pmatrix}=\begin{pmatrix}x\\3y-x\end{pmatrix}$ then the values of x and y are ......

a. $x=2,\ y=1$ b. $x=1,\ y=-1$ c. $x=1,\ y=1$ d. $x=1,\ y=2$

Given $A=\begin{bmatrix}a&b&c\\d&e&f\\g&h&i\end{bmatrix}$ with $\det(A)=7$

9. $\begin{vmatrix}a+d&b+e&c+f\\2(a+d)&2(b+e)&2(c+f)\\-3g&-3b&-3i\end{vmatrix}=$.........

a. -42 b. 42 c. 0 d. 7

10. $\begin{vmatrix}2a-d&2b-e&-4c+2f\\-g&-h&2i\\-d&-e&2f\end{vmatrix}=$.........

a. 28 b. -42 c. 42 d. -28

Part II Answer each of the following questions in the space provided. Explain and show your work.

1) Given the matrix $A = \begin{pmatrix} 2 & 4 & 0 \\ 1 & 1 & -1 \\ -1 & 3 & 0 \end{pmatrix}$.

a. Calculate the determinant of A using the method of columns

(5 pts)

$$\begin{pmatrix} 2 & 4 & 0 \\ 1 & 1 & -1 \\ -1 & 3 & 0 \end{pmatrix} \begin{matrix} 2 & 4 \\ 1 & 1 \\ -1 & 3 \end{matrix}$$

0   -6   0

0   4   0

10

b. Consider the system $\begin{cases} 4x_2 + 2x_1 = 20 \\ x_2 - x_3 + x_1 = 2 \\ -x_1 + 3x_2 = 0 \end{cases}$ Use Cramer's Rule to find **only** x_2.

(7 pts)

$$\begin{pmatrix} 2 & 4 & 0 \\ 1 & 1 & -1 \\ -1 & 3 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 20 \\ 2 \\ 0 \end{pmatrix}$$

$$x_2 = \frac{|A_2|}{|A|} = \frac{\begin{vmatrix} 2 & 20 & 0 \\ 1 & 2 & -1 \\ -1 & 0 & 0 \end{vmatrix}}{10} = \frac{+20}{10}$$

$$= 2$$

2) Given $A = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}$ and $C = \begin{pmatrix} 3 & 2 \\ 8 & 5 \end{pmatrix}$, Find a matrix B such that $(A - 2I + 4B)^T = C^{-1}$.

(6 pts)

$$A - 2I + 4B = \begin{pmatrix} -5 & 2 \\ 8 & -3 \end{pmatrix}^T$$

$$4B = \begin{pmatrix} -5 & 8 \\ 2 & -3 \end{pmatrix} - \begin{pmatrix} -1 & 2 \\ 4 & -5 \end{pmatrix} = \begin{pmatrix} -4 & 6 \\ -2 & 2 \end{pmatrix}$$

$$B \begin{pmatrix} -1 & 3/2 \\ -1/2 & 1/2 \end{pmatrix}$$

3) Construct the 3×3 lower triangular matrix A whose non zero entries a_{ij} are

(6 pts)

$$a_{ij}=\begin{cases} i+j & \text{if } i\neq j \text{ and } i \text{ is even} \\ i^2-2j & \text{if } i\neq j \text{ and } i \text{ is odd} \\ i+2j & \text{if } i=j \end{cases}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

lower triangular so $a_{12}=0$, $a_{13}=0$, $a_{23}=0$

$a_{11} = 1+2\times1 = 3$

$a_{22} = 2+2\times2 = 6$

$a_{33} = 3+2\times3 = 9$

$a_{21} = 2+1 = 3$

$a_{31} = 3^2-2\times1 = 9-2 = 7$

$a_{32} = 3^2-2\times2 = 9-4 = 5$

$$\begin{pmatrix} 3 & 0 & 0 \\ 3 & 6 & 0 \\ 7 & 5 & 9 \end{pmatrix}$$

4) Given the matrices $A=\begin{pmatrix} 3 & 1 & 2 \\ 0 & x & 1 \\ 2 & 1 & 0 \end{pmatrix}$ and $B=\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$

(7 pts)

a. Use the method of cofactors to determine in terms of x the determinant of A

If we use the 1^{st} column

$|A| = 3(x\times0-1\times1) - 0 + 2(1\times1-2\times x)$

$= 3(-1)+2(1-2x) = -3+2-4x = -1-4x$

b. Determine the value of x so that $\det(2A)=\det(B)$

$|2A| = 2^3|A| = 8(-1-4x)$

$|B| = 2\times3\times(-2)\times(-2) = 24$

$|2A| = |B|$

$8(-1-4x) = 24$

$-1-4x = 3$

$-4x = 4$

$x = -1$

$S=\{-1\}$

5) Let $A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & -1 & 2 & -1 \\ 1 & -2 & -1 & 2 \\ 2 & 1 & -2 & -1 \end{pmatrix}$. Find AA^T. Then determine the value of the determinant of A.

(8 pts)

$$AA^T = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & -1 & 2 & -1 \\ 1 & -2 & -1 & 2 \\ 2 & 1 & -2 & -1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & -1 & -2 & 1 \\ 1 & 2 & -1 & -2 \\ 2 & -1 & 2 & -1 \end{pmatrix} = \begin{pmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{pmatrix}$$

$|AA^T| = 10^4 \rightarrow |A|\,|A^T| = 10^4$ but $|A| = |A^T|$

$|A|\,|A| = 10^4$

$|A|^2 = 10^4$

6) Given $A = \begin{pmatrix} 3 & 1 & 2 \\ -1 & 2 & x \\ 3 & -2 & 1 \end{pmatrix}$ Determine the value of x so that the matrix of cofactors will be

(6 pts)

$A_c = \begin{pmatrix} 10 & 13 & -4 \\ -5 & -3 & 9 \\ 0 & -14 & 7 \end{pmatrix}$

In the matrix A, the cofactor c_{11} is

$$c_{11} = (-1)^{1+1} M_{11} = M_{11} = \begin{vmatrix} 2 & x \\ -2 & 1 \end{vmatrix} = 2+2x$$

In the matrix A_c we have $c_{11} = 10$

Therefore $2 + 2x = 10 \rightarrow 2x = 8 \rightarrow x = 4$

7) Consider the matrix $A = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 2 & 1 \\ -1 & -3 & 3 \end{pmatrix}$. Find A^{-1} using the Gaussian elimination.

(20 pts)

Then use it to solve the system: $\begin{cases} x_1 + 2x_2 + x_3 = x_3 - 1 \\ x_1 + x_3 = 2 - 2x_2 \\ x_3 + 3x_2 + x_1 = 2 + 4x_3 \end{cases}$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ -1 & -3 & 3 & 0 & 0 & 1 \end{array}\right) \quad -r_1 + r_2$$

$\times -1$, $+$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ -1 & -3 & 3 & 0 & 0 & 1 \end{array}\right) \quad r_1 + r_2$$

$\times 1$, $+$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & -1 & 3 & 1 & 0 & 1 \end{array}\right) \quad -r_3 \text{ and interchange } r_2 \text{ and } -r_3$$

$\times -1$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & -1 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}\right) \quad -2r_2 + r_1$$

$+$, $\times -2$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 6 & 3 & 0 & 2 \\ 0 & 1 & -3 & -1 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}\right) \quad 3r_3 + r_2$$

$+$, $\times 3$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 6 & 3 & 0 & 2 \\ 0 & 1 & 0 & -4 & 3 & -1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}\right) \quad -6r_3 + r_1$$

$+$, $\times -6$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 9 & -6 & 2 \\ 0 & 1 & 0 & -4 & 3 & -1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}\right)$$

A^{-1}

$$\begin{cases} x_1 + 2x_2 + x_3 = x_3 - 1 \\ x_1 + x_3 = 2 - 2x_2 \\ x_3 + 3x_2 + x_1 = 2 + 4x_3 \end{cases}$$

$$\begin{cases} x_1 + 2x_2 = -1 \\ x_1 + 2x_2 + x_3 = 2 \\ -x_1 - 3x_2 + 3x_3 = -2 \end{cases}$$

$$X = A^{-1}B$$

$$= \begin{pmatrix} 9 & -6 & 2 \\ -4 & 3 & -1 \\ -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} -25 \\ 12 \\ 3 \end{pmatrix}$$